CHAPTER 4: STUDIES OF ESTIMATION

4.0 PROBLEMS ASSOCIATED WITH THE LITERATURE SEARCH

In many of the studies to be discussed in this section, academically more able subjects have been studied to determine how people should estimate. This technique, suggested by Ginsburg et al [1983], allows one to gain insight into strategies available to pupils by determining what techniques or strategies are used by `experts' in the field. I find this slightly problematic as this implies there are correct ways of estimating and that the academically `weaker' pupils must do it along the same lines. It also relies heavily on the researcher's access to `good' estimators, in this case. I was more interested in discovering how most pupils estimate - not just the `good' ones. I also wanted to discover the aspects of estimation that were most problematic to most pupils and to try to understand the reasons for these difficulties.

Until recently, estimation has not been part of an agreed curriculum. Even now, with its inclusion in the National Curriculum, it is often seen as a `tacked on' topic and, I fear, not valued by the pupils. Most estimation tasks in school require an estimate and then (almost immediately) a measure or calculation is made. Many colleagues have agreed with me when I have asserted that pupils often make their `estimate' **after** they have measured or calculated showing their disregard for the estimation process.

During the course of the literature search, I found many articles describing the benefits of estimation and methods or strategies employed by subjects when estimating. I did not find a great quantity of research in which the actual estimates of the subjects were readily available. The few research studies which did include data about the number of subjects `succeeding' in a set of estimation tasks used different criteria for determining the reasonableness of the answers and consequently, it was impossible to compare these studies to any large degree. I wrote to each of the researchers who had included data requesting further information but unfortunately none of the original data was available for me to use the same criterion on all data.

I also found that transferability across the various estimation tasks did not take place readily according to the research that was available. In other words, a `good' computational estimator was not necessarily a `good' quantitative estimator.

I shall describe each of the studies found in the literature and attempt to show how these linked together but, for the aforementioned reason, sometimes this proved impossible.

I have found estimation difficult to analyze. Perhaps it is because it is such a private activity and difficult for subjects to relate how they are doing it. This problem has been a persistent one and has been the main impediment in many lines of the research. When pupils state that their estimate was "Just a Guess", it may be any number that came in their head or it may be that they have a very good intuitive feel for the number. The literature search was useful in providing the means to interpret findings and to show approaches to the problem by other researchers and it also gave support for my findings of the problematic nature of estimation.

4.1 INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) devoted the 1986 Yearbook to "Estimation and Mental Computation". Many pleas for teaching estimation are included in this book but the final article states "Little research has been devoted to the teaching and learning of estimation." [Benton 1986, p246] Another problem which has been identified by Sowder [1992] is that investigators do not necessarily agree on the most important research issues nor on how research should proceed nor on how estimation, number sense and mental computation should be incorporated into the curriculum. There has also been a lack of agreement as to how to proceed with research and also how to incorporate the work into the classroom. Case [1989] discussed the relationship of the epistemological position of the researcher and its relevance to their work in developing children's number sense. Of the three epistemological traditions of empiricism, rationalism and socioculturalism, I find myself to be a rationalist and it is from that perspective that this document is written.

Generally, the studies are easily divided between the areas of computational and quantitative estimation and this chapter will be so divided. However, one of the earliest works in the area of estimation dealing with both areas was Paull's [1971] doctoral dissertation. He tested 196 pupils, aged sixteen, in college preparatory classes from an upper middle class community in New York. One of his conclusions was that the ability to estimate is not a unitary ability. In other words, estimation tasks of different types i.e. numerical estimates and computational estimates appear to require different abilities and there does not appear to be an easy transfer of ability between the various estimation tasks. The diversity of estimation tasks and performance of these tasks is a recurring theme in the research.

4.2 COMPUTATIONAL ESTIMATION

Reys, et al [1982] conducted a study in which 1200 people (across the age-range 12 year-old to adults) took a computational estimation test. Reys has contributed a great deal to the literature in this field and this study is one of the most extensive I found in the literature search. All pupils were in `the upper track' classes and the adults (mostly college graduates) were successful in their careers. The top 10% of each age group on the test paper were deemed to be `good estimators' and 59 of these subjects were interviewed.
I find great difficulty in accepting these `good estimators' as such, as some of these individuals believed their estimate to be incorrect when faced with calculator or computer output which disagreed with the estimate. This may also give evidence of the lack of transferability across estimation tasks.

Various processes which `good estimators' used in estimating have been described by Reys, et al [ibid] as determined by the interview technique. These were found to be high-level cognitive processes that are difficult to define operationally. Their attempt at these definitions and the terminology used by Reys, et al follow:

Reformulation alters the numerical data into a more mentally manageable form. Thirty percent of 440,000 becomes 1/3 of 450,000. The structure of the problem remains intact.

Translation changes the structure of the problem into a more mentally manageable form. An example of this might be to estimate total school attendance for a week by multiplying a `reasonable' daily average by 5 rather than adding 5 numbers together.

Compensation is a further process which adjusts the estimate at various stages in the estimation process. How much would each person receive, roughly, if a pools syndicate of 23 people won a total of ,221 876? The estimator might alter the total to ,230 000 to allow an easy division and then `compensate' the answer to give a final estimate of ,9 000.

Characteristics of the good estimators were identified. Quick and accurate mental computation and recall of basic facts and an understanding of place value were present in each subject interviewed and they all utilised **Reformulation**. Each had a `Tolerance for Error' which one seventh grade (12 year old) pupil explained to be the ability to "tell yourself not to be bothered by being off some" [ibid, p198]. A majority of the good estimators used **Translation** and **Compensation**. A minority, 20-50% of the `good estimators' interviewed (considered by Reys, et al to be on a higher `level'), revealed that they searched through a variety of strategies e.g. (Averaging and Compensating) before proceeding and they felt a strong sense of self-confidence in their estimating ability. One confounding issue identified in this study is that the characteristics of good estimators have three distinct dimensions: number skills, cognitive processes and affective attributes, and each of these is not dichotomous but distributed along a continuum [ibid]. Further support of this statement is given by the conference report from San Diego where Resnick [1989] claimed that number sense (which is closely linked to estimation) is nonalgorithmic and involves judgment and interpretation, the application of multiple criteria, self-regulation of the thinking process, and imposed meaning. She also claims it tends to be complex and effortful. The `nonalgorithmic' nature of estimation links nicely with reformulation, translation and compensation which have definite attributes of `number skills'. One can include `the application of multiple criteria' and `self regulation of the thinking process' as `cognitive processes'. It is important to recognise, as an estimator, that your estimate will be `wrong' but the willingness to impose one's own criterion upon the estimate and remain in control of the rigour of one's estimate might be an example of this aspect. Finally, the fact that it is complex and effortful and requires judgment and interpretation place it clearly within the affective domain and all of these aspects are distributed along a continuum. The continuous nature of these aspects deserve some explanation. At one extreme of the number skills attribute, one can imagine the mathematically able individual with considerable skills in recognising a variety of means of expressing a number through the darts player able to calculate scores mentally with great accuracy until one reaches the individual with very poor number skills in any environment. Examples of the range of cognitive processes are more difficult but it can be seen that simply restricting this to the application of a single criterion at one extreme through applying a multiplicity of criteria at the other gives a `flavour' of the continuous nature of this aspect. Finally, the affective attribute might range from the very confident and effort-making individual through the confident but `lazy' to eventually the `lazy' and insecure individual. I shall discuss the links I have found between confidence and estimation in Chapter 10.

Another study reported by Levine [1982] determined the computational estimation ability of 89 undergraduates - none of whom were mathematics majors - and the strategies used by them which were classified into various categories. The categories had been predetermined through a literature search, pilot testing, and a logical examination of the computational estimation process. An article based on this study described how each subject was given a test and asked to "think aloud". The categories (with an illustrative example, where necessary) were:

1) Fraction (0.76 becomes :)
2) Exponents (0.047 becomes 5×10^{-2})
3) Rounding both numbers
4) Rounding one number
5) Powers of 10 (76 x 89 becomes 100 x 100)
6) Known numbers (27.2 x 4.63 becomes 25 x 4)
7) Incomplete Partial Products(Quotients)
   (689 x 34 becomes 600 x 30 + 90 x 4)

8) Proceeding Algorithmically

The strategy types found to be used most frequently were **Rounding Both Numbers** and **Proceeding Algorithmically**. The Proceeding Algorithmically strategy was exemplified by a subject estimating 64.6 x 0.16 by computing
646 x 6 = 3876 or 3000, 646 x 10 = 6460 or 6000, adding these results to get 9000, and replacing the decimal point for a final estimate of 9. In my opinion, the widespread use of this `strategy' indicates a paucity of adequate strategies available to the subjects interviewed (though one reason for its use may have been that the test did not have time constraints). It should be remembered that these subjects were not selected for their estimation ability as was the case in the Reys, et al study. Levine [ibid] stated that the ability to estimate was not found to be related in her study to the number of different strategies used. This finding appears to show disagreement with those of Reys, et al. This apparent lack of consistency will be seen again between other studies. Dowker discussed her findings at a British Society for Research into Learning Mathematics (BSRLM) meeting utilising Levine's test with a group of 35 professional mathematicians. "Each answer was given a score from 0 to 3 depending on the percentage error." Her main finding was that "the mathematicians used a very great variety of strategies" in determining their estimates. [Dowker 1988, p3] This would, therefore, appear to show agreement with the Reys, et al study but an element of disagreement with the findings of Levine.

A study to understand how the affective domain impinges on estimation ability was made by Bestgen, et al [1980]. A group of 187 preservice elementary teachers were divided into three groups to help ascertain how attitudes and estimation ability could be altered by a scheme of instruction over a ten week period. One group of 50 were the control group, another group of 79 received weekly practice quizzes, and the third group of 58 took these quizzes but also were taught specific estimation strategies. Attitude instruments were designed to determine the teachers' attitude towards estimation. Pre-treatment and post-treatment tests and attitude instruments were given to each group. The results of the study showed that practice improved performance and when the "practice was accompanied with specific estimation strategies, greater understanding and respect for estimation processes occurred." [ibid, p135]

Levine [1980] reported, in her doctoral dissertation, that a further strategy called the Establishing Bounds Strategy was identified during the pilot study but not used by subjects in the final study. I believe this strategy to be a very important one. The recognition that is implicit when bounds for the estimate are given shows that the estimator knows s/he is not expected to get the answer `right'. The wording of a question requesting **an** estimate, however, implies one number and this is a possible explanation for the absence of this strategy in her final study. Levine [ibid] did not identify a reason for the absence of this strategy in the main study but it may be that less emphasis was placed on **an** estimate in the pilot study. I shall discuss a technique for encouraging pupils to develop the Establishing Bounds strategy in Chapter 10.
Levine [ibid, p7] identified skills, which had been deduced by an analysis of the task, to be valuable in the estimation process; these are listed below:

1) Understanding of place value,
2) Ability to round numbers,
3) Ability to multiply/divide by powers of ten,
4) Ability to add, subtract, multiply and divide two numbers which are powers of ten,
5) Understanding of basic operations and how numbers behave under them, and
6) A feeling for the fractional equivalents of decimals.

The skills identified are those which are normally associated with mathematically able pupils. Reinforcement of this claim comes from Paull [op cit] who found that computational estimates were positively correlated with mathematical ability and verbal ability.

Rubenstein [1985, p107] investigated the responses to four types of estimation tasks; her label for these "Estimation Scales", an example of each and the means of assessment are given below:

1) "Open-ended" - Estimate the cost of a collection of priced items - Allowed interval of answers determined by a panel of mathematics educators,
2) "Reasonable vs. Unreasonable" - Is the calculator display of 324 x 6 shown, right?, wrong? or don't know - Display was exact or outside the bounds of the open-ended interval but within a power of ten of the correct answer,
3) "Reference Number" - Is the difference between $55.65 and $26.95 above or below $20 (don't know was another possible answer) - The reference number was one of the bounds of the open-ended acceptable interval,
4) "Order of Magnitude" - Is 74.4) 24 closest to 0.3, 3, 31 or 310? - Correct option was within the open-ended acceptable interval.

She [ibid pp109-110] also developed a multiple-choice "Related Factors Test" to measure the mathematical skills listed below (with an example, where necessary):
1) "Selecting Operations",
2) "Making Comparisons", Which is larger? 39 x 29, 29 x 41 or are they equal?
3) "Knowing Number Facts",
4) "Operating with Tens", 64.8) 10 = 648, 64.80, 64.8 or 6.48
5) "Operating with Multiples of Tens",
6) "Knowing Place Value",
7) "Rounding",
8) "Judging Relative Size", Choose the largest number 9.8, 9.08, 9.80, or 9.9.

She found that "operating with tens, making comparisons and judging relative size" were the most important contributory factors in the prediction of estimation performance but that "knowing place value, knowing number facts, rounding, and operating with powers of ten - did not explain more than 1% of the variance in any stepwise regressions." [ibid p117] Sowder [1992 p375] suggests that Rubenstein's `good predictors' might be too closely associated with place value and basic number understanding "for all to be significant in a stepwise regression." These results show some of the problems associated with the research into estimation.

Shoen et al [1981] conducted a teaching study to ascertain the possibility of improving the computational estimating ability of primary-aged pupils. The study found that they could be taught to become better estimators of whole number computations but several problems were identified during the study. Some pupils in the study knew an exact answer through rapid mental computation, but rounded it to get an estimate. This makes a mockery of estimation and may have developed through estimating being confused with rounding but it indicates that estimation was not valued as a process by the pupils. Many pupils were found to think "of estimating as a process to be learned and used separate from computation" [ibid, p177]. These statements show the possibility that many pupils do not understand what is required when asked to give an estimate. Dowker has made several contributions to research in estimation and presented findings at BSRLM conferences. She has worked with pupils in the lower age range restricted to computational estimation tasks. Her method involved dividing the pupils (aged 5 - 9 years) into sets on the basis of their performance on a set of addition problems. These different sets were assigned `Levels' and each set was given estimation sums "that were just a little too difficult for them to solve correctly." [Dowker 1989, p7] They also were given sums that were of much greater difficulty and the results showed that pupils scored reasonably well on the problems near their own Level but poorly on problems that were much higher than their own Level. This work concentrates on a lower age range, in general, than other researchers. It has not been very helpful to my study primarily due to the restriction in the age range and the tasks themselves.

Sowder and Wheeler [1989] investigated the development of concepts and processes which they determined to be associated with computational estimation. Their analysis considered the major factors affecting computational estimation to be broadly grouped in the following components (which will be explained in the following paragraph):

1) Conceptual Components,
2) Skill Components,
3) Related Concepts and Skills, and
4) Affective Components.

The Conceptual Components recognised the role of approximate numbers, the potential for a multitude of possible techniques and outcomes, and finally, the appropriateness of an estimate was subject to context and desired accuracy. The Skill Component closely mirrored the processes described by Reys et al [1982] previously mentioned. Related Concepts and Skills were similar to those found by Levine [1980 & 1982] and the Affective Components included recognition of the usefulness of estimation, tolerance of error and confidence in mathematical ability and ability to estimate.

Sowder and Wheeler [op cit] were particularly interested in the first two components in the above list. They described to pupils situations in which computational estimates were required together with some hypothetical pupils' explanations. The `subject' pupils were then questioned about these hypothetical pupils' explanations. Finally, some open response problems were given where the pupils were required to give estimates for a set of arithmetic problems.

Several disturbing findings are given in their report. Pupils were found to prefer "computing with the exact number and then rounding off the result to obtain an estimate" [ibid p143] This shows how estimation is not highly regarded by pupils and it was found that "the preference for this process grew in direct proportion to the development of the ability to estimate by the standard round-then-compute method." [ibid p144] Few pupils would accept the validity of a number of possible outcomes for the same problem. Finally, the "rounding process used by many students, however, was school learned and algorithmic." [ibid] It would appear that the pupils considered there to be only one `right' answer to an estimation problem and applied a rule which had been given to them in school. It should be clear that teachers will need to be aware of the need to avoid teaching the techniques of computational estimation as yet another set of rules.

4.3 QUANTITATIVE ESTIMATION

Several studies have been conducted to determine the cognitive factors which affect a subject's ability to estimate quantity. Immers [1983], in his doctoral dissertation, studied the linear estimation ability of primary school pupils. He was interested in the strategies employed as they made their estimates. He found that the most frequently used strategy to estimate length was **Unit Iteration**; some pupils did this physically but most did so visually by marking off units with their eyes.[1] He found that estimation of measures could be effectively taught to primary-aged pupils, although accuracy with a single unit was not found to be directly transferable to other units and, in novel situations, pupils tended to underestimate length.[ibid] Again, this indicates the lack of transferability of estimation tasks.

A study by Corle described the difficulties middle school pupils had in making estimates and then measuring various quantities. An Editor's note stated, "The enormity of pupils' errors, both in estimating and in actual measurement, when dealing with some of the most common quantities, concepts upon which our textbook problems are based, certainly accounts for some of the pupils' troubles in problem solving, but leaves us wondering how we are spending our time" [Corle 1960, p340]. I have heard similar complaints from colleagues when pupils have been asked questions about heights or lengths of everyday objects. If they estimate the height of a double decker bus to be 20 metres or more, it shows they have great difficulty with either the metric system or the notion of height.

High school seniors were studied by Crawford and Zylstra in 1952 to attempt to correlate measures of achievement and estimating ability as determined by a test they had devised. The measures of achievement were taken to be performance on the Otis Self Administering

Mental Ability Test[2], the Schorling-Clark-Potter Hundred Problem Arithmetic Test[3], the pupil's mathematics marks, and total grade point

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[1] In his summary of Piagetian Research on Measurement Concept Attainment, he claimed the age for conservation of length to be between 6 and 8. He also claimed that this is "the pervasive step that must be achieved before meaningful measurement can occur"[Immers 1983, p11].

[2] According to F. Kuder [1975, p331], Professor of Psychology ,Duke University, Durham, N.C., the Otis Test "compares favorably with other measures of general ability."

[3] A review by Betz [1949, p432], Specialist in Mathematics, Public Schools, Rochester, N.Y. stated that "items appear to have been carefully selected. Technicalities are avoided: only basic techniques are stressed. Hence the test as a whole may be regarded as a satisfactory instrument of

averages. They found "the ability of pupils to estimate with competency is not related to their mental abilities, computational abilities or marks received in mathematics courses" [Crawford & Zylstra 1952, p246].

Their criterion for `correct' on the estimating test calls for some comment. The test was multiple-choice with the `correct' answer given and four other answers. A conclusion that was made from the data is that overall, pupils were "poor estimators of exact measurements (the `correct' answer) but when nearly correct or approximate answers are allowed for, their performance is somewhat more satisfactory" [ibid, p242]. Two items from the test given in the paper showed answers deemed incorrect to be between 40 and 70 per cent above or below the correct answers. As other studies are analyzed, these figures shall be seen to be important. In my opinion, the lack of consistency in the criterion for `correctness' in estimation studies is a major problem.

The primary and middle school age range and a group of adults were studied by Siegel, et al [1982] in an attempt to develop a model of estimation on the basis of their performance. They utilised rational task analysis[4] and pilot test data to propose a competence model which distinguished between **Benchmark** and **Decomposition/Recomposition** estimation tasks. Their term Benchmark includes that of Unit Iteration (previously discussed) but also includes **Fractional Benchmark** and **Multiple Benchmarks**. `How thick is a strand of spaghetti?' might be a Fractional Benchmark question. The estimator, in this case, must choose a suitable unit (possibly the cm.) and then decide what part of that unit would be a reasonable estimate for the strand of spaghetti. An example given by Siegel, et al for a Decomposition/Recomposition task is, "How wide is the car park where I left my car?". The problem is `decomposed' into an estimate of how many parking spaces along one edge, the width of each space, and then recomposed to determine the width of the car park.

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evaluation."

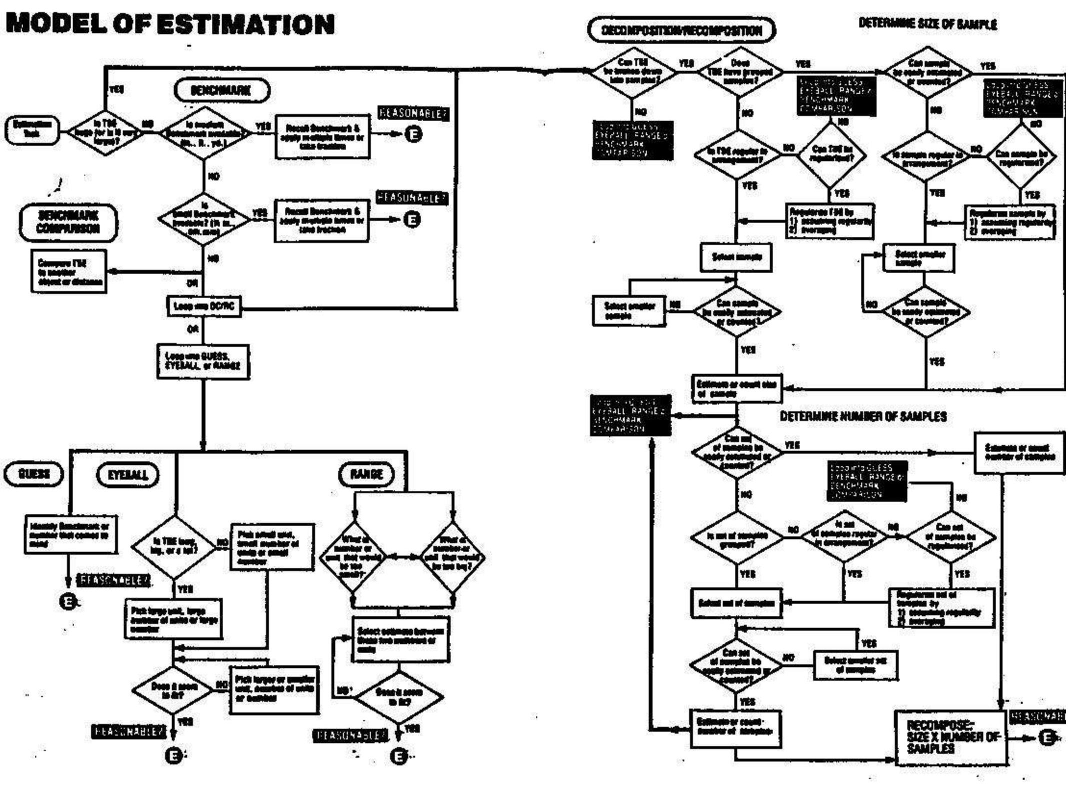
[4] The task which the subject must perform is subdivided into various subtasks which are hierarchically sequenced.

Ten boys and ten girls randomly selected at each grade level from 2 to 8 inclusive participated in the study. Ten adults, six women and four men, were included and their educational level ranged from BA to PhD. Questions were developed which were found to be of interest to the pupils through pilot testing and an explanation of the purpose of the test and a definition of an estimate was given. The pupils were interviewed on a tape recorder and probes were made when clarification was deemed necessary. Adults wrote both answers and explanations of how their estimates were made. The questions developed were pre-classified into problem types. These were Benchmark-Extent, Fractional Benchmark-Extent, Regular Decomposition-Extent and Numerosity, and Irregular Decomposition-Extent and Numerosity. Extent refers to linear extent, i.e. length or height. An example of Irregular Decomposition would be to estimate the number of cars in a large car park with random empty spaces. Answers were judged to be `Reasonable' if within one order of magnitude e.g. incorrect by no more than a power of ten and `Accurate' if within ±50% of the actual value.

In this study, the terms problem type and strategy were often used interchangeably. When the pupil estimated the length of an object by relating it to another object, the pupil was said to be using the Benchmark Comparison Strategy. Other `strategies' e.g `Guess' or `Eyeball' were defined before the test. The strategies used by the pupils were determined through interview. Each strategy, with a brief explanation, is given below:

1. Don't know - No explanation given
2. Guess - Just thought of an answer
3. Eyeball - Perceptual description
4. Range - Indicated bounds
5. Benchmark Comparison - Compared stimulus to another object
6. Benchmark - As above but with mentally carried unit
7. Fractional Benchmark - Fractional part of 6 above
8. Multiple Benchmark - Multiples of 6 above
9. Pseudo-Decomposition - Showed awareness that the problem could be broken down but did not use technique
10. Decomposition/Recomposition - Successful in breaking problem into parts and recombining

A result of this study was to enable Siegel, et al to propose a hierarchy of estimating tasks. Benchmark-Extent was found to be the easiest (e.g. How long is this pencil?) and Irregular Decomposition/Recomposition Extent (e.g. What is the length of this reel of computer tape?) and Numerosity (e.g. How many holes in the head of a tennis racquet?) were the most difficult. These were not unexpected results. However, the study showed the proposed model in need of revision. A `final' proposed model is given in Figure 4.1 below.



Revised model of the estimation process.

The classification of the various tasks by Siegel, et al is useful and shows a variety of potential strategies. It is interesting to note that Siegel's model is rather obvious from the point of view that it is a logical sequencing of types of strategies. However, if a subject is correct within a given criterion and states that s/he is "Guessing" or "Eyeballing", is it correct to say that this answer is less valid than that given by a person who uses "Decomposition/Recomposition"? It is obvious that the Decomposer/Recomposer is using mathematical strategies more overtly but the intuitive guesser who is `right' may be using these same strategies or even more powerful ones at an innate level and these are inaccessible to the researcher. I certainly feel that sometimes I was unable to discover how very good estimates were made in the course of my own work.

Again the problem of comparability arises with, for example, the study by Crawford and Zylstra as the criterion for `accuracy' is dissimilar. `Reasonable' to Siegel, et al is incorrect to Crawford and Zylstra and some of Siegel's `accurate' answers would also be incorrect to Crawford and Zylstra. This exemplifies the problem of consistency previously mentioned.

Bright conducted a study on a small number (21) of experienced teachers. He found that they lacked skill in various types of estimation tasks but found that estimating lengths improved with practice. An important part of this study was Bright's [1979a, p161] attempt to determine "if estimation skills can be developed for real situations, is there corollary development in estimating measurements given solely in symbolic form?" In a test for determining the ability to select the greater of two measurements in symbolic form, the subject was shown two objects labelled A and B and then asked "Which would be larger, 2A or 4B?". In other words, the subject could be shown a pencil (A) and a key (B) and then asked, what would be larger, two pencils or four keys? He found that there does not appear to be a relationship between estimating physical measurements and choosing the greater of two measurements in symbolic form. This prompted Bright [ibid p163] to suggest "all types of estimation should be presented to students in order to develop skills in working with as many types as possible".

Using a correct criterion of ∀15%, Attivo [1979, p18], in her doctoral dissertation, found that transfer did not appear in a study of prospective teachers' ability to estimate lengths and areas and, consequently, she concluded that "instruction is needed in most types of estimation". This is a recurring recommendation in many studies of estimation as transfer between various tasks appears limited and yet, it would appear beneficial if pupils could develop the means for transfer. However, until pupils are more accustomed to estimating and valuing their estimates, we cannot expect them to be able to make the transfer to novel situations.

Hildreth [1983], in an article based upon his doctoral dissertation, discussed strategies used by pupils aged 10, 12, and 18 years when estimating length and area. Each pupil was given a test of perceptual ability (a Group-Embedded Figures Test). The mathematical achievement of the 10 and 12 year old pupils was based on their Scholastic Aptitude Tests in Mathematics and those of the 18 year old pupils was found using a locally developed minimum competency test. In his article, Hildreth described some invalid strategies that pupils have been found to use. Wild guessing is an obvious one and used for length and area. Many pupils, according to Hildreth [ibid], used a `count-around' or `count-around plus some for the middle' method when finding the area of a shape; or they may just add the length and width together. These strategies indicate the pupils display a misunderstanding of area. This may be caused by the request for speed in making their estimate. A strategy that has not already been identified, for estimating length, was that of prior knowledge. The fact that ceiling tiles are one square foot and a foot is about 30cm allows the estimator to determine the length of the room in metres. Appropriate strategies for area estimates were: repeated addition (a type of unit iteration of an area), length times width, and rearrangement - to obtain a shape easier to estimate. An inappropriate strategy found by Piaget in studies of primary grade pupils and reported by Hildreth [ibid] was that of `Centering', where the pupil gives the estimated length of one of its dimensions to be the area of the shape. Again, this is an inappropriate strategy that may be occurring due to the requirement of speed. Hildreth observed each subject answering a set of 24 questions and noting the estimation strategy which was used on a form containing 9 appropriate strategies e.g.(L x W), 7 inappropriate methods e.g. `Centering' or Other Methods. All of the Other Methods were found to be inappropriate. Hildreth [1980] reported in his doctoral dissertation the calculation of a strategy use score (STRUSE) obtained by subtracting the number of items in which an inappropriate strategy was used from 24, the number of questions. He then used the STRUSE score as a means of correlating strategy use with various other factors.

Hildreth's [ibid] study included a teaching programme in which half of the ten and twelve year-olds were taught to use guess and check methods and the other half were given explicit instruction in appropriate length and area estimation strategies. He found, as a result of a pre-test and post-test, that the pupils improved in estimation ability and those who had been taught strategies showed an improvement in strategy use.

Hildreth [ibid] observed that the strategies used by the pupils were not qualitatively different from those used by the young adults. Also, the estimating ability and strategy use were correlated to the tested perceptual ability. Estimation ability was determined using a `correct' criterion of relative error e.g. absolute error/true value less than one-third. It was positively correlated with strategy use but neither sex nor grade level were found to be factors in estimating ability or strategy use. An interesting finding was that estimation ability and strategy use were found to be related to mathematical ability for the young adults but unrelated for the pupils. This contradicts Paull's [op cit] findings that 16 year-old pupil's ability to estimate length was not correlated with either mathematical ability or verbal ability, nor was the ability to estimate length or area correlated with the ability to do mathematical computations rapidly. The Crawford and Zylstra [1952] study of 18 year-olds agreed with Paull's [1971] findings. A possible reason for the apparent contradictions is the differences in the criteria for correctness.

4.4 SUMMARY

There are encouraging signs of a growing interest in research in this field and some studies have produced useful results. Studies conducted have shown the strategies used by `good' computational estimators are consistent with a mathematical analysis of the task i.e. compensation is valid on mathematical grounds for altering an original estimate. Other studies have shown there is a great deal more that must be done before an understanding of the estimation process is achieved. It has been suggested that affective attributes, e.g. self-confidence, may be contributory factors in computational estimation ability. Sowder [1992, p379] generated profiles of good estimators as people who have "strong self-concepts in mathematics, attribute success to ability, have too little experience with failure to attribute it to any cause, and value mental computation and estimation." The profiles were not universally true but they may be an indicator. Affective aspects of estimation will be investigated in a future chapter.

Training studies of quantitative estimation have shown that there is a lack of transfer between estimation tasks involving different units and situations with the plea for all types of estimation to be practised in the classroom.

There have been inconsistencies in various studies linking `mathematical ability' with estimation ability. I believe these inconsistencies may have several causes. First of all, the criteria for correctness are not consistent between studies. Since the raw data is not available from any of the studies, it is impossible to determine if the inconsistencies are caused by this factor. I wrote to each of the researchers where data was mentioned only to find that none of the original data could be found. Secondly, the tests for `mathematical ability' were not identical. And thirdly, the estimation tasks set may have been so different as to confound the possibility of agreement between studies. Hildreth [1980] suggested, in his doctoral dissertation, that a standardised estimation ability test be developed and agreement reached upon whether relative error scores or some other scores should constitute estimation ability.
I agree with this suggestion. It may be difficult to develop a standardised estimation test but I certainly believe that a consistent means of assessing estimation answers would be an advantage.

It is interesting to note that most of the studies conducted have involved subjects in the higher academic ability range. Pre-service teachers, `upper track' classes, undergraduates, elementary teachers, adults with BA to PhD, and pupils from college preparatory classes were mentioned in many of the studies. The primary age pupils were subjects of three studies, i.e., Dowker, Immers and Shoen et al. The only broad-based studies noted were those by Edwards, Siegel et al, and Hildreth. Little data was available from the Edwards study. Consequently, the only studies to cover a cross-section of the population and provide data are Siegel's and Hildreth's.

I believe an important area of estimation which has not been studied is that of how the vast majority of people actually estimate and what causes their failure, if they do fail. In an effort to understand this area, I conducted a research study culminating in the development of a test for primary and secondary school pupils in the London Borough of Sutton. The next two chapters will discuss this work.