CHAPTER 3: THE VALUE OF ESTIMATION

3.0 INTRODUCTION

Several publications have pronounced that the skill of estimation is of importance to mathematics education. The Cockcroft Report included it in its Foundations List devoting paragraphs 257-262 to the topic but declared that "estimation is not practised in very many classrooms" [Cockcroft 1982, para 258]. A report by H.M.I. [DES 1985, p18] included the "ability to estimate" as `Objective 15'. Finally, the National Curriculum [DES 1991b] specifically identifies estimation and approximation in almost every level of Attainment Target 2.

I will discuss how the need for developing the skill of estimation has been encouraged prior to these three documents and consider reasons for teaching estimation in the previously defined areas of computational and quantitative estimation. Recently, a conference in San Diego encouraged the inclusion of estimation in the mathematics curriculum for two basic reasons; firstly, it helps develop conceptual structures for number and secondly, it is useful in everyday life [Sowder 1989].

3.1 COMPUTATIONAL ESTIMATION

Recent pleas to teach computational estimation are not the first to be made and, in my opinion, unless estimation is more clearly understood, its recent inclusion in the National Curriculum will not stimulate understanding of estimation by pupils in school. Opinions have been expressed by various writers on the importance of developing estimation skills. Complaints that `approximation' was badly taught were registered in the reports of the London University on the General School Examinations in 1928-30 as reported by Inman [1932] in the Mathematical Gazette. His use of `approximation' here is very close to the definition given heretofore for computational estimation. In 1932, he advocated scrapping rules on decimal placement in long multiplication and division [ibid]. The rules he refers to are those of counting decimal places in multiplier and multiplicand to determine the decimal placement in the product and the similar method for long division decimal placement. He wanted to avoid the compartmentalising of `approximation' to special lessons on the subject. Sauble [1959, p33], a Detroit schoolteacher, claimed in 1955 that pupils who succeed in estimating do not always employ the standard algorithms but "develop ingenuity and resourcefulness in dealing with numbers" and that estimation stimulates a more mature understanding of basic principles of the structures involved. She called for estimation to be part and parcel of arithmetic instruction and not something separate.

Buchanan [1980] claimed that estimation could be used as a part of learning about mathematical structure and developing a `sense' of number. The structures Buchanan and Sauble refer to include the commutative, transitive, and associative properties. For example, the early stages of understanding commutativity may be shown by the following example:

> If a is a `large' number and b is a (relatively) `small' number, the importance of a is obvious regardless of whether the addition is a + b or b + a.

I admit this is an early stage in the development and the major advantage to the `estimating pupil' should be in his/her improved number sense. Edwards, A [1984, p61], who has developed a programme for computational estimation in Papua New Guinea, claimed that it is the "most direct, and obvious way of teaching" the concepts and skills involved in `number sense'. Edwards defined `number sense' as a shorthand for a `general quantitative literacy' and the capacity to compare numbers. These rather vague terms can be clarified best by an example. `Number sense' allows one to understand, among other things, that 3470 is close to 3500 while 46 is not close to 16 although the difference in each case is 30. Edwards A. [ibid] suggested four reasons for the lack of estimation in the curriculum:

1) the multiplicity of methods prevent the subject being treated simply,
2) there are no right or wrong answers,
3) assessment is consequently difficult and,
4) pupils and some teachers view it as `lazy'.

Finally, computational estimation allows a check on the reasonableness of any calculated answer. This ability will become more and more important as the use of electronic calculating aids becomes more widespread. Reys (a prolific writer on this subject) et al [1980] stated, however, that estimation skills do not automatically develop through maturation or from the study of more mathematics. They point out that `good estimators' can fall prey to believing a calculator over the estimated answer "claiming they must have made some mistake" [ibid, p231]. This is a source of concern and will be discussed in the next chapter.

O'Daffer [op cit, p48], a Professor of Mathematics involved in teacher education, claimed that estimating can "help to develop a positive attitude towards mathematics since the pupils are not competing with each other to get the right answer, but rather, are competing with themselves to improve the accuracy of their estimations". He claimed that it also provides motivation for solving problems and that once pupils commit themselves to a guess, they will often attempt to find the exact answer.

3.2 QUANTITATIVE ESTIMATION

Are there similar advantages to be gained by teaching quantitative estimation? Bell M. [1974] maintained that an essential part of every person's mathematical education is the ability to use estimation and approximation confidently, and readily. He suggested that this included `number sense', `measure sense' and an awareness of reasonable cost or amounts.

Estimation is a vital part of the measurement process. Before we start to measure anything, we need to select a unit. This selection involves our estimate of the magnitude of the attribute to be measured. When pupils are learning about measurement, they often have a notion of exact measurement. I found, during my seventeen years of teaching in secondary schools, that many pupils believed exact measurements are possible and just a matter of improved skill by the measurer. Pupils are sometimes told that a line is exactly 4cm long. Later in their education, they are told that this really is not true and that no measurement can be exact. Precision is obviously important but a more honest approach could be to develop the notion that measurements are given with an element of error. Trimble [1973, p133] indicated that using an interval of values "provides a mathematical model for the arithmetic of measurement that contrasts with the rather sloppy, this-works-even-though-it-isn't-really - right rubrics with which we currently mystify children". Pupils should not be misled into the idea of `exactness' that can never be substantiated. Personal experience has shown that pupils will mark their own work wrong if they are `off' a measurement value given in the answer book by the slightest amount.

Estimating may be useful when some topics are developed. For example, a starting point for angle measurement builds on the pupil's understanding of half and quarter turns. The notion of greater precision may be introduced and, consequently, the need for a measurement using a protractor or angle indicator. The `estimating' pupil should not make the common error of recording the supplementary angle. Other areas of the syllabus arise naturally from estimation. Payne and Seber [1959] claimed that estimates can be useful in teaching angle but also they give a natural entry to percentages. Quite interesting problems of permissible error on a given `measurement' could develop into a topic for sixth form students.

Quantitative estimation can also lead to topics which could `stretch' the most able pupils. Van de Walle and Thompson suggested that `uncountable' sets such as grains of sand, drops of water, etc. could be estimated. This could be done by a form of counting, i.e., sampling, weighing, etc. This could allow some pupils to gain a type of perception of thousands and millions. Van de Walle and Thompson [1985, p8] also suggested introducing a note of levity to lessons by asking "How many steps would an ant take to walk around the building?"

Finally, quantitative estimation can help the pupil to describe the world s/he occupies. Bright [1976, p103] asserted "Amount (e.g., length, mass, temperature) is an important part of one's understanding of the world. Estimation skill can help organise this world so that patterns can be observed."

3.3 ESTIMATION IN PROBLEM SOLVING

Many writers (Bruner, Polya, Biggs, Burton) have stated that a major purpose of studying mathematics is to enable one to solve problems. In the past, pupils have typically been taught a great deal of mathematical content and were then expected to be able to use this to solve problems. Mathematics educators, such as Schoenfeld [1986], have suggested that a way to teach mathematics is to help pupils to solve problems and to use the problem as a motivator for developing mathematical content. The great problem-solver, G. Polya [1981, p156], described the scientific method as "Guess and Test" and suggested that teachers should encourage pupils to guess with the exhortation: "Let us teach guessing!" [Polya 1954, p158] Polya does not consider this to be an easy task but one which is most worthwhile. Estimating, as defined earlier, may be an important tool in the problem-solver's repertoire. In particular, I claim it is a vital element to the problem-solving heuristic of Guess and Test and, possibly, `Make the Problem Simpler'. When the problem-solver makes the problem simpler, one aspect of this is to guess which parts of the problem can be `ignored'. Another way of stating the same thing is that the problem-solver estimates which are the vital parts of the problem.

3.4 SUMMARY

Opinions of writers as to the benefits of teaching computational estimation to the pupil have been put forward. It can be seen that a wide body of professional opinion supports the claim that computational estimation is both important and valuable in the development of mathematical education. The benefits, if substantiated, surely make estimation skills highly desirable. If they are so desirable, why is there so little evidence of them being developed?

Sowder [1992, p387] claims that computational estimation is closely related to number sense and that number sense is difficult to define and therefore difficult to assess. Perhaps this is another reason for estimation not having had a high profile in the mathematics curriculum.

It has also been indicated that quantitative estimation may be an important element in measurement and in allowing the pupil to gain a greater understanding of his/her environment.

Finally, it has been proposed that estimation skills are closely linked with problem solving heuristics. The `estimating pupil' should have more `feeling' for the magnitude of an answer whether it is a number or a measure and s/he may be more willing to risk experimenting in mathematics and potentially gaining greater insight and intuition in his/her own mathematical development. This may all be wishful thinking but it would appear that there are no major disadvantages inherent in estimating provided the pupil recognises the need for some level of accuracy for given tasks.