CHAPTER 2: THE NATURE OF ESTIMATION

2.0 INTRODUCTION

"An estimate is required!" What is meant by this request? The word `estimate' may convey a variety of meanings to people in their place of work. When the car body shop produces an `estimate' for repair work, the amount of work which they have on their books may influence their estimate. Statisticians, on the other hand, use the term statistical estimation for the process of combining or reducing data to produce a statistic called an estimate, according to Lindgren [1975]. These two `estimates' may involve precise written calculations with the results quoted to quite a high degree of accuracy. An estimate as referred to in the National Curriculum involves checking that calculations and measurements are reasonable [DES 1991b]. The terms estimation, mental arithmetic, and approximation are presently used almost interchangeably in mathematics education literature. What, then, is meant by these terms? Before proceeding further, I will discuss the use of these various terms and I will develop a working definition of estimation.

The term, mental arithmetic, is often confused with estimation. Atweh [1982] has divided arithmetic into four categories in an article in the 1982 National Council of Teachers of Mathematics Yearbook. His divisions are based upon whether the method of calculation is mental or uses some concrete aid, e.g. pencil and paper, and upon the precision of the result. He uses the term estimate as meaning an inexact solution. The four categories are:

Category 1 Concrete-Exact,
Category 2 Mental-Exact,
Category 3 Concrete-Estimate, and
Category 4 Mental-Estimate.

"Find the product of 87.2 and 21.9" may be a useful example to clarify his divisions.

Category 1 would be, in this case, a long multiplication problem or using a calculator.

Category 2 would, in the case of the example, be the amazing skill illustrated by some people in performing this calculation mentally but would also include the more mundane calculations performed by the average barperson in determining the cost of a round of drinks.
Category 3 gives an inexact answer using pencil and paper, e.g. 90 x 20 = 1800.
Category 4 would be the same calculation performed mentally and is often called estimation according to Atweh [1982 p53].

Bright [1979b, p581], whose contribution in the area of quantitative estimation has been considerable, stated that: "Understanding estimation is important for understanding measurement. Every measurement is an approximation, or if you will, an estimate." Kyburg & Smokler [1964, p52], experts in probability, stated that "all practical values are only approximate".

The Collins English Dictionary [1988, p500] defines an estimate as "an approximate idea of distance, cost, size, etc; calculate roughly". An approximation can be considered to be "a result precise enough for a specific purpose" according to Hall, L. [1984, p517] (of Richmond Public Schools, Richmond, Virginia). Her definition differs from that of Schmid [1967 p366], a principal in Baltimore, Maryland, who defines approximation as "a judgment (as distinguished from a wild guess) of a magnitude or of the size of an object or of a group of objects". He also defines estimation "to be a computation (emphasis mine) involving approximated numbers or quantities" [ibid p367]. Hall, L. [op cit, p517] defines estimation to be "the mental skill of making an educated guess". Williamson [1978], from the University of Wisconsin, defines approximation as the process of performing a `rough' calculation with the given data and estimation as the process of performing a `calculation' without all the given data.

It is possible to see from the previous paragraphs that there is a variety of opinion as to what constitutes an approximation and what constitutes an estimate with the added confusing term, mental arithmetic. Mental arithmetic will be defined, for the purpose of this document, as any arithmetical operation performed without concrete aids. Approximation will be defined as the process of determining the value of a measurable quantity but without the aid of a measuring instrument. I prefer Hall's definition of estimate ("the mental skill of making an educated guess") but would wish to modify it to include the skill of making an educated guess using pencil and paper, where necessary. A revised definition of estimation incorporating the Collins Dictionary's and that of Hall would read as follows: "The skill of making an educated guess as to the value of a distance, cost, size, etc., or arithmetic calculation". An estimate, then, would be the `educated guess'. It will be seen from the above definitions that the set of mental arithmetic skills and approximation will intersect those of estimation. These definitions are consistent with the use of these terms in the National Curriculum [DES 1991b].

It was important for me to keep the definitions for estimation, approximation and mental arithmetic firmly in mind when reading other writer's work. These definitions are consistent with those of the other researchers in the field.

2.1 TYPES OF ESTIMATION

I will discuss the various ways in which estimation as defined can be classified. **Computational estimation** gives a rapid `rough' answer to an arithmetic problem such as:

$$\frac{6.49 \times 98.73}{81.7}$$

Another category, **numerical estimation**, indicates the number of objects in a collection. This could be an estimate of the number of people in a crowd as estimated by the Metropolitan Police. The third type, **quantitative estimation**, indicates the size (length, weight, volume) of an object or group of objects. This could be an estimate of the length of a train in metres. This type of estimation would include estimations of time such as: how many minutes will it take to walk to the train station? It should be recognised that a continuous quantity e.g. length and a discrete quantity e.g. number of people are both, nonetheless, quantities. Ellis [1968, p159] states that counting may be called a measuring procedure but he makes the point that "it is unique among all measuring procedures". His reason for expressing this uniqueness is that there is no arbitrary standard against which one is measuring as in all other types of measuring. I intend to consider the numerical estimate and the quantitative estimate together and will incorporate them into the single term, quantitative estimation. This redefinition will be useful as some techniques for determining the number of items in a collection utilise techniques developed for estimating the length, area, or volume. It will also be shown that to perform quantitative estimation, often the estimator may wish to perform computational estimation as well.

2.1a COMPUTATIONAL ESTIMATION

`Experts' utilise various techniques, which will be discussed in Chapter 4, when performing computational estimation. These techniques, however, share a commonality and can all be considered arithmetical in nature. For example, an estimate for 63.2 x 1.47 will involve less sophisticated techniques than an estimate for the problem shown below:

$$\frac{9.84^2 \times (6.73 - 2.9)^3}{9.87}$$

The estimator in the first example will possibly mentally calculate 60 x 1 and 60 x 2, and then average these answers to obtain 90 as the estimate. In the second case, the estimator might find an estimate for the cube of the bracket as 60 utilising the knowledge that the difference between the numbers is less than 4 and that $4^3 = 64$, therefore, roughly 60. The estimator may then `cancel' a 9.84 with the 9.87 and then perform a final estimate by multiplying 10 x 60 giving 600 as the estimate. Although these problems vary greatly in complexity, they are similar in type.

2.1b QUANTITATIVE ESTIMATION

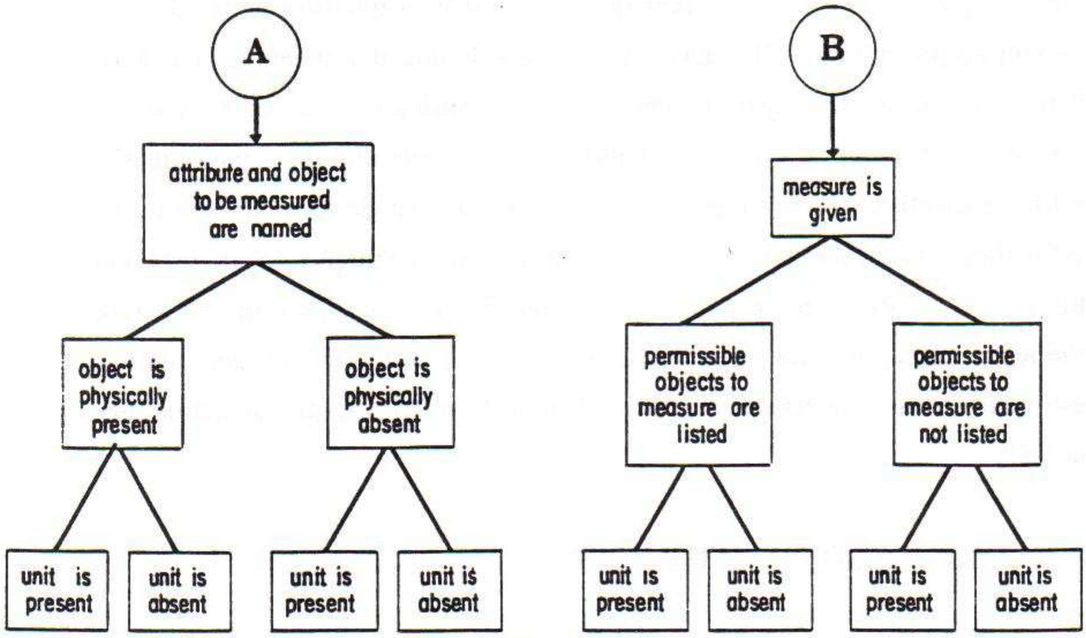
When one considers quantitative estimation, there are complications. First of all, certain aspects of measurement must be developed before the estimator can start. The notion of quantity (except when dealing with numerical estimates) implies a means of measuring that quantity and therefore a unit of measure is necessary. The size of the unit has a direct bearing on the magnitude of the measurement, i.e. a small unit will yield a larger magnitude for the measurement than a larger unit would yield. Also, any measurement must be considered to be independent of its position in space, i.e. the measurer must practice conservation before s/he will be able to measure. It should be recognised that measurement is important in quantitative estimation and every measurement has limits to its accuracy as no measurement can be exact.[1] Bakst [1937] states that measurements are always approximate because no instrument is perfect and people make errors in measuring.

As has been noted, numerical estimation is being regarded as a subset of quantitative estimation. There are also various task types of quantitative estimation. For example, the task of estimating the length of rope stretched between two pegs while one holds a metre rule in the hand (as a guide) is different from performing the same task with only a mental image of that unit. George Bright [1976] classified quantitative estimation into eight types and the reader is referred to the diagram of the model shown in Figure 2.1 below.

[1] According to Heisenberg's Uncertainty Principle, when one tries to measure an object at the atomic level (absolute precision), the act of measuring influences the particle one is trying to measure. A complete discussion of this can be found in most textbooks on Nuclear Physics.

FIGURE 2.1

THE 'BRIGHT MODEL'



Kinds of estimation

First, he divided all tasks into two classes after he has assumed that the unit of measure had been selected for each class. By naming the unit, Bright may have eased the task if the estimator did not have any idea of the object's size. For class A estimates, the estimator guesses the measure for a named attribute of an object plus the named unit i.e. the length of a rope in metres. It could also be argued that another class exists which would be class A but without a named unit or even a named system, eg metric or Imperial. Chapter 9 will show the manner in which I used this particular `sub-class' which proved quite illuminating. I did not consider this further complication to be valuable in understanding the estimation process in the early stage of the research. My reasoning at the time was that the process was already quite complicated and I knew that I would have to develop some means of assessing estimation ability. I could be faced with answers that would be difficult to process. For example, if asked for the height of the tree, a subject could reply, "twice the height of the garage". I did not consider this omission from Bright's model to be significant but it did remove the possibility of obtaining potentially useful information.

For class B estimates, the estimator chooses an object to which a specified measure could be assigned i.e. what object has a mass of 2 kg. Two subclasses of class A are defined by whether or not the estimator can see the object to be estimated. For class B, the two subclasses are determined by whether a list of permissible objects is given or not. Finally, each of the four subclasses are further divided into two types, by whether or not the estimator has the unit of measurement physically present.

O'Daffer [1979] stated that there are three aspects of any quantitative measurement:

1) an entity to be measured,
2) a unit of measure, and
3) a number called the measure.

The suggestion is made that estimating takes place when two of the three aspects are given and the third is required but cannot be easily determined. This fits well into Bright's model. The Bright model will be used in future chapters to classify tasks. A certain amount of control is kept in the hands of the researcher using this model and this may be a valid criticism of the model. It has limitations but it does allow many of the tasks generally associated with quantitative estimation to be categorised.

2.2 SUMMARY

Definitions of important terms and a brief description of the various types of estimation have been given. A system for classifying quantitative estimation tasks has been presented. However, classification of estimation tasks is not the only area of interest to me. Strategies for computational and quantitative estimation will be the focus of Chapter 4 describing studies of estimation. Before that discussion, I intend to show that estimation is an important area in mathematical education.